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# ON A REDUCTION OF NON-COMMUTATIVE REIDEMEISTER TORSION FOR HOMOLOGY CYLINDERS

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## 1. INTRODUCTION

Let $\Sigma_{g,1}$ be a compact oriented surface of genus $g \geq 1$ with one boundary component.

Homology cylinders over a surface were first introduced by Goussarov [4] and Habiro [6] in their surgery theory of 3-manifolds developed for the study of finite-type invariants. In [3, 9] Garoufalidis and Levine introduced the homology cobordism group of homology cylinders, which can be seen as an enlargement of the mapping class group of the surface. We denote by $C_{g,1}$, $C_{g,1}^{irr}$ and $\mathcal{H}_{g,1}$ the monoid of homology cylinders over $\Sigma_{g,1}$, the submonoid consisting of irreducible ones as 3-manifold and the smooth homology cobordism group respectively. The Johnson filtrations $C_{g,1}[k]$, $\mathcal{H}_{g,1}[k]$ of $C_{g,1}$, $\mathcal{H}_{g,1}$ are defined as the kernels of the actions on $\pi_1\Sigma_{g,1}/(\pi_1\Sigma_{g,1})_k$, where the lower central series $G_k$ of a group $G$ is defined inductively by $G_1 :=$ $G$ and $G_{k+1} := [G_k, G]$.

Sakasai [15, 16] studied torsion invariants of homology cylinders with in general non-commutative coefficients and showed by the degrees of these invariants associated to elements of $H^1(\Sigma_{g,1})$ as a reduction that the submonoids $C_{g,1}^{irr} \cap C_{g,1}[k]$ for $k \geq 2$ and $\mathrm{Ker}(C_{g,1} \to \mathcal{H}_{g,1})$ have abelian quotients isomorphic to $(\mathbb{Z}_{\geq 0})^\infty$. Note that since the connected sum of a homology cylinder and a homology 3-sphere is another homology cylinder, it is reasonable to restrict our attention to $C_{g,1}^{irr}$ in considering "size" of $C_{g,1}[k]$. Morita [12] showed by using his "trace maps" defined in [11] that the abelianization of $\mathcal{H}_{g,1}[2]$ has infinite rank. Goda and Sakasai [5] showed by using sutured Floer homology theory that $C_{g,1}^{irr}$ has an abelian quotient isomorphic to $(\mathbb{Z}_{\geq 0})^\infty$. Cha, Friedl and Kim [1] showed by using abelian torsion invariants that the abelianization of $\mathcal{H}_{g,1}$ contains a direct summand isomorphic to $(\mathbb{Z}/2)^\infty$, and that the abelianization of $\mathcal{H}_{g,1}[2]$ contains a direct summand isomorphic to $(\mathbb{Z}/2)^\infty$ and one isomorphic to $\mathbb{Z}^\infty$ if $g \geq 2$.

The aim of this note is to present another reduction of non-commutative torsion invariants introduced in [8] and to give another approach to Sakasai's result for $C_{g,1}^{irr} \cap C_{g,1}[k]$. More precisely, we consider the coefficients of the maximum order terms of torsion invariants associated to bi-orders of $\pi_1\Sigma_{g,1}/(\pi_1\Sigma_{g,1})_k$ and use them to prove that the group completion of $C_{g,1}^{irr} \cap C_{g,1}[k]$ has an abelian group quotient of infinite rank for $k \geq 2$. In [8] we can find an analogous work on submonoids of $C_{g,1}^{irr}$ associated to solvable quotients of $\pi_1\Sigma_{g,1}$.

In this note all homology groups and cohomology groups are with respect to integral coefficients unless specifically noted.

## 2. HOMOLOGY CYLINDERS

First we recall the definitions of homology cylinders and their homology cobordisms. See [7], [17] for more details on homology cylinders.

To simplify notation we often write $\Sigma, \pi$ instead of $\Sigma_{g,1}, \pi_1\Sigma_{g,1}$, respectively. We take a base point for $\pi$ in $\partial\Sigma$.

**Definition 2.1.** A *homology cylinder* $(M, i_\pm)$ over $\Sigma$ is defined to be a compact oriented 3-manifold $M$ together with embeddings $i_+, i_-: \Sigma \to \partial M$ satisfying the following:
(i) $i_+$ is orientation preserving and $i_-$ is orientation reversing,
(ii) $\partial M = i_+(\Sigma) \cup i_-(\Sigma)$ and $i_+(\Sigma) \cap i_-(\Sigma) = i_+(\partial M) = i_-(\partial M)$,
(iii) $i_+|_{\partial\Sigma} = i_-|_{\partial\Sigma}$,
(iv) $(i_+)_*, (i_-)_*: H_*(\Sigma) \to H_*(M)$ are isomorphisms.
Two homology cylinders $(M, i_\pm), (N, j_\pm)$ are called isomorphic if there exists an orientation preserving homeomorphism $f: M \to N$ satisfying $j_\pm = f \circ i_\pm$. We denote by $C_{g,1}$ the set of all isomorphism classes of homology cylinders over $\Sigma_{g,1}$.

A product operation on $C_{g,1}$ is given by stacking:

$$(M, i_\pm) \cdot (N, j_\pm) := (M \cup_{i_- \circ (j_+)^{-1}} N, i_+, j_-),$$

which turns $C_{g,1}$ into a monoid. The unit is given by the standard cylinder $(\Sigma\times[0,1], id\times1, id\times0)$.

As pointed out in [5, Proposition 2. 4] there is an epimorphism $F: C_{g,1} \to \theta^3$ as follows, where $\theta^3$ is the monoid of homology 3-spheres with the connected sum operation. For $(M, i_\pm) \in C_{g,1}$, we can write $M = M' \sharp M''$, where $M'$ is the prime factor of $M$ containing $\partial M$. Then $F(M, i_\pm) := M''$. Therefore it is reasonable to consider the submonoid $C_{g,1}^{irr}$ consisting of all homology cylinders whose underlying 3-manifolds are irreducible.

**Definition 2.2.** Two homology cylinders $(M, i_\pm) \sim_m (N, j_\pm)$ are said to be *homology cobordant* if there exists a compact oriented smooth 4-manifold such that:
(i) $\partial W = M \cup_{i_+ \circ j_+^{-1}, i_- \circ j_-^{-1}} (-N)$,
(ii) $H_*(M) \to H_*(W), H_*(N) \to H_*(W)$ are isomorphisms.
We denote by $\mathcal{H}_{g,1}$ the quotient set of $C_{g,1}$ with respect to the equivalence relation of homology cobordism.

The monoid structure of $C_{g,1}$ naturally induces a group structure of $\mathcal{H}_{g,1}$. The inverse of $[M, i_\pm] \in \mathcal{H}_{g,1}$ is given by $[-M, i_\mp]$.

We set $N_k := \pi/\pi_k$. For $(M, i_\pm) \in C_{g,1}, (i_\pm)_*: N_k \to \pi_1 M/(\pi_1 M)_k$ are isomorphisms according to Stallings' theorem [18]. We define a homomorphism $\varphi_k: C_{g,1} \to \operatorname{Aut} N_k$ by $\varphi_k(M, i_\pm) := (i_+)_*^{-1} \circ (i_-)_*$. By abuse of notation we also denote by $\varphi_k$ the naturally induced homomorphism $\mathcal{H}_{g,1} \to \operatorname{Aut} N_k$.

**Definition 2.3.** The *Johnson filtrations* of $C_{g,1}$ and $\mathcal{H}_{g,1}$ are the sequences

$$\cdots \subset C_{g,1}[k] \subset \cdots \subset C_{g,1}[2] \subset C_{g,1}[1] = C_{g,1},$$
$$\cdots \subset \mathcal{H}_{g,1}[k] \subset \cdots \subset \mathcal{H}_{g,1}[2] \subset \mathcal{H}_{g,1}[1] = \mathcal{H}_{g,1}$$

respectively, where $C_{g,1}[k]$ and $\mathcal{H}_{g,1}[k]$ are the kernels of $\varphi_k$.

## 3. A reduction of the torsion homomorphism

Next we review torsion invariants of homology cylinders and introduce another reduction of the group where these torsion invariants are defined, using a bi-order of the nilpotent quotient $N_k := \pi/\pi_k$.

Let $\mathbb{K}$ be a skew field. We write write $\mathbb{K}_{ab}^\times$ for the abelianization of the unit group $\mathbb{K}^\times$. For a finite CW-pair $(X, Y)$ and a homomorphism $\rho: \mathbb{Z}[\pi_1 X] \to \mathbb{K}$ such that the twisted homology group $H_*^\rho(X, Y; \mathbb{K})$ associated to $\rho$ vanishes, the Reidemeister torsion $\tau_\rho(X, Y) \in \mathbb{K}_{ab}^\times / \pm\rho(\pi_1 X)$ associated to $\rho$ is defined. See [10] and [19] for more details.

We set $A_k := \pi_{k-1}/\pi_k$. Since $A_k$ is torsion-free for all $k$, $N_k$ has a finite filtration of normal subgroups such that all successive quotient are torsion-free abelian groups. It is known that for such a group $G$ (called poly-torsion-free-abelian), $\mathbb{Q}[G]$ is a right (and left) Ore domain; namely $\mathbb{Q}[G]$ embeds in its classical right ring of quotients $\mathbb{Q}(G) := \mathbb{Q}[G](\mathbb{Q}[G] \setminus 0)^{-1}$ [13]. For $(M, i_\pm) \in C_{g,1}$, we denote by $\rho_k$ the composition of the homomorphisms

$$\mathbb{Z}[\pi_1 M] \to \mathbb{Z}[\pi_1 M/(\pi_1 M)_k] \xrightarrow{(i_+)_*^{-1}} \mathbb{Z}[N_k] \to \mathbb{Q}(N_k).$$

See [2, Proposition 2. 10] for a proof of the following lemma.

**Lemma 3.1.** *For $(M, i_\pm) \in C_{g,1}$, $H_*^{\rho_k}(M, i_+(\Sigma); \mathbb{Q}(N_k)) = 0$.*

**Definition 3.2.** We define a map $\tau_k \colon C_{g,1} \to \mathbb{Q}(N_k)_{ab}^\times / \pm N_k$ by

$$\tau_k(M, i_\pm) := \tau_{\rho_k}(M, i_+(\Sigma)).$$

The following proposition is a version of [1, Proposition 3. 5] and [8, Corollary 3.10]. See also [16, Proposition 6. 6] for a related result. The proof is almost same as that of [8, Corollary 3.10], and so we omit the proof.

**Proposition 3.3.** *The map $\tau_k \rtimes \varphi_k \colon C_{g,1} \to (\mathbb{Q}(N_k)_{ab}^\times / \pm N_k) \rtimes \mathrm{Aut}(N_k)$ is a homomorphism.*

**Corollary 3.4.** *The map $\tau_k \colon C_{g,1}[k] \to \mathbb{Q}(N_k)_{ab}^\times / \pm N_k$ is a homomorphism.*

We denote by $\bar{\cdot} \colon \mathbb{Z}[N_k] \to \mathbb{Z}[N_k]$ the involution defined by $\bar{\gamma} = \gamma^{-1}$ for $\gamma \in N_k$ and naturally extend it to $\mathbb{Q}(N_k)$. We set

$$D_k := \{\pm\gamma \cdot q \cdot \bar{q} \in \mathbb{Q}(N_k)_{ab}^\times \, ; \, \gamma \in N_k, q \in \mathbb{Q}(\Gamma_m)_{ab}^\times\}.$$

The following theorem is also a version of [1, Theorem 3. 10] and [8, Corollary 3.13]. See them for the proof.

**Theorem 3.5.** *The map $\tau_k \rtimes \varphi_k \colon \mathcal{H}_{g,1} \to (\mathbb{Q}(N_k)_{ab}^\times / D_k) \rtimes \mathrm{Aut}(N_k)$ is a homomorphism.*

**Corollary 3.6.** *The map $\tau_k \colon \mathcal{H}_{g,1}[k] \to \mathbb{Q}(N_k)_{ab}^\times / D_k$ is a homomorphism.*

A *bi-order* $\leq$ of a group $G$ is a total order of $G$ satisfying that if $x \leq y$, then $axb \leq ayb$ for all $a, b, x, y \in G$. A group $G$ is called *bi-orderable* if $G$ admits a bi-order. It is well-known that every finitely generated torsion-free nilpotent group is residually $p$ for any prime $p$. Rhemtulla [14] showed that a group which is residually $p$ for infinitely many $p$ is bi-orderable. Together with the fact that $N_k$ is torsion-free, we see that $N_k$ is a bi-orderable.

In the following we fix a bi-order of $N_{k-1}$. We define a map $c \colon \mathbb{Z}[N_k] \setminus 0 \to \mathbb{Q}(A_k)^\times / \pm A_k$ by

$$c\left(\sum_{\delta \in N_{k-1}} \sum_{\gamma \in N_k, [\gamma] = \delta} a_\gamma \gamma\right) = \left[\left(\sum_{\gamma \in N_k, [\gamma] = \delta_{max}} a_\gamma \gamma\right) \gamma_0^{-1}\right],$$

where $\delta_{\max} \in N_{k-1}$ is the maximum with respect to the fixed bi-order such that for some $\gamma \in N_k$ with $[\gamma] = \delta_{max}$, $a_\gamma \neq 0$, and $\gamma_0 \in N_k$ is an element with $[\gamma_0] = \delta_{max}$. The proof of the following lemma is straightforward.

**Lemma 3.7.** *The map $c \colon \mathbb{Z}[N_k] \setminus 0 \to \mathbb{Q}(A_k)^\times / \pm A_k$ does not depend on the choice of $\gamma_0$ and is a monoid homomorphism.*

By the lemma we have a group homomorphism $\mathbb{Q}(N_k)^\times_{ab}/\pm N_k \to \mathbb{Q}(A_k)^\times/\pm A_k$ which maps $f\cdot g^{-1}$ to $c(f)\cdot c(g)^{-1}$ for $f,g \in \mathbb{Z}[N_k]\setminus 0$. By abuse of notation, we use the same letter $c$ for the homomorphism. Since there is a natural section $\mathbb{Q}(A_k)^\times/\pm A_k \to \mathbb{Q}(N_k)^\times_{ab}/\pm N_k$ of $d$, $\mathbb{Q}(A_k)^\times/\pm A_k$ can be seen as a direct summand of $\mathbb{Q}(N_k)^\times_{ab}/\pm N_k$.

For irreducible $p,q \in \mathbb{Z}[A_k]\setminus 0$, we write $p\sim q$ if there exists $a\in A_k$ such that $p = \pm a\cdot q$. Since $\mathbb{Z}[A_k]$ is a unique factorization domain, every $x\in\mathbb{Q}(A_k)^\times/\pm A_k$ can be written as $x=\prod_{[p]}[p]^{e_{[p]}}$, where $e_{[p]}$ is a uniquely determined integer. We have an isomorphism $e\colon \mathbb{Q}(A_k)^\times/\pm A_k \to \oplus_{[p]}\mathbb{Z}$ defined by $e(x)=\sum_{[p]}e_{[p]}$. Thus we obtain a homomorphism $e\circ c\circ\tau_k\colon C_{g,1}[k]\to\oplus_{[p]}\mathbb{Z}=\mathbb{Z}^\infty$.

## 4. Construction and computation

In this section we systematically construct the images of $e\circ c\circ\tau_k\colon C_{g,1}[k]\to\mathbb{Z}^\infty$.

For nontrivial $\gamma\in\pi$ and a tame knot $K\subset S^3$, we construct a homology cylinder $M(\gamma,K)$ as follows. Let $*\in\Sigma$ be the base point for $\pi$. We choose a smooth path $f\colon[0,1]\to\Sigma$ representing $\gamma$ such that $f^{-1}(*)=\{0,1\}$, and define $\tilde{f}\colon[0,1]\to\Sigma\times[0,1]$, $h\colon[0,1]\to\Sigma\times[0,1]$ by $\tilde{f}(t)=(f(t),t)$ and $h(t)=(*,1-t)$. After pushed into the interior, $\tilde{f}\cdot h$ determines a tame knot $J\subset \mathrm{Int}(\Sigma\times[0,1])$. Let $E_J$ be the complement of an open tubular neighborhood $Z$ of $J$. We take a framing of $J$ so that a meridian of $J$ represents the conjugacy class of the generator of the kernel of $\pi_1\partial Z\to H_1(\Sigma\times[0,1])$ compatible with the orientation of $J$ and that a longitude of $J$ represents the conjugacy class of the image of $\gamma$ by $(i_-)_*\colon\pi\to\pi_1E_J$. Let $E_K$ be the exterior of $K$. Now $M(\gamma,K)$ is the result of attaching $-E_K$ to $E_J$ along the boundaries so that a longitude and a meridian of $K$ correspond to a meridian and a longitude of $J$ respectively.

**Lemma 4.1.** *For all nontrivial $\gamma\in\pi$ and all knots $K\subset S^3$, $M(\gamma,K)\in C^{irr}_{g,1}\cap(\cap_k C_{g,1}[k])$.*

*Proof.* If $K$ is a trivial knot, then $M(\gamma,K)$ is the unit of $C_{g,1}$ for all nontrivial $\gamma\in\pi$, and there is nothing to prove. In the following we assume that $K$ is nontrivial.

Since $E_J$ and $E_K$ are both irreducible and $\partial Z$ and $\partial E_K$ are both incompressible, $M(\gamma,K)$ is also irreducible.

Extending a degree 1 map $(E_K,\partial E_K)\to(Z,\partial Z)$ by the identity map on $E_J$, we have $f\colon M(\gamma,K)\to\Sigma\times[0,1]$. The following commutative diagram of isomorphisms shows that $M(\gamma,K)\in C_{g,1}[k]$ for all $k$:

$$\begin{tikzcd}
 & \pi_1M(\gamma,K)/(\pi_1M(\gamma,K))_k \arrow[dd, "f_*"] & \\
N_k \arrow[ur, "(i_+)_*"] \arrow[dr, "(i_-)_*"'] & & N_k \arrow[ul, "(i_+)_*"'] \arrow[dl, "(i_-)_*"] \\
 & \pi_1(\Sigma\times[0,1])/(\pi_1(\Sigma\times[0,1]))_k &
\end{tikzcd}$$

□

**Proposition 4.2.** *Let $\gamma\in\pi_k\setminus 1$. Then $\tau_{k+1}(M(\gamma,K))=[\Delta_K(\gamma)]$ for all $K$.*

*Proof.* We have the following short exact sequences of twisted chain complexes:

$$0\to C^{\rho_k}_*(\partial E_K)\to C^{\rho_k}_*(E_J,i_+(\Sigma))\oplus C^{\rho_k}_*(E_K)\to C^{\rho_k}_*(M(\gamma,K),i_+(\Sigma))\to 0,$$

$$0\to C^{\rho_k}_*(\partial Z)\to C^{\rho_k}_*(E_J,i_+(\Sigma))\oplus C^{\rho_k}_*(Z)\to C^{\rho_k}_*(\Sigma\times[0,1],i_+(\Sigma))\to 0,$$

where all the coefficients are understood to be $\mathbb{Q}(N_k)$. It is easily checked that

$$H_*^{\rho_k}(\partial E_K; \mathbb{Q}(N_k)) = H_*^{\rho_k}(E_K; \mathbb{Q}(N_k)) = H_*^{\rho_k}(\partial Z; \mathbb{Q}(N_k)) = H_*^{\rho_k}(Z; \mathbb{Q}(N_k)) = 0.$$

Therefore by the homology long exact sequences

$$H_*^{\rho_k}(E_J, i_+(\Sigma); \mathbb{Q}(N_k)) = 0.$$

Considering multiplicativity of Reidemeister torsion in the above exact sequences we obtain

$$\tau_{\rho_k}(E_J, i_+(\Sigma)) \cdot \tau_{\rho_k}(E_K) = \tau_{\rho_k}(\partial E_K) \cdot \tau_{\rho_k}(M(\gamma, K), i_+(\Sigma)),$$
$$\tau_{\rho_k}(E_J, i_+(\Sigma)) \cdot \tau_{\rho_k}(Z) = \tau_{\rho_k}(\partial Z) \cdot \tau_{\rho_k}(M(id), i_+(\Sigma)).$$

Here

$$\tau_{\rho_k}(E_K) = [\Delta_K(\gamma)(\gamma - 1)^{-1}],$$
$$\tau_{\rho_k}(Z) = [(\gamma - 1)^{-1}],$$
$$\tau_{\rho_k}(\partial E_K) = \tau_{\rho_k}(\partial Z) = \tau_{\rho_k}(\Sigma \times [0,1], i_+(\Sigma)) = 1,$$

which are easy to check. Now these equations give the desired formula. □

Recall that for every monoid $S$, there exists a monoid homomorphism $g\colon S \to \mathcal{U}(S)$ to a group $\mathcal{U}(S)$ satisfying the following: For every monoid homomorphism $f\colon S \to G$ to a group $G$, there exists a unique group homomorphism $f'\colon \mathcal{U}(S) \to G$ such that $f = f' \circ g$. By the universality $\mathcal{U}(S)$ is uniquely determined up to isomorphisms. Finally, using the homomorphism $e \circ c \circ \tau_k\colon C_{g,1}[k] \to \mathbb{Z}^\infty$, we give another proof of the following theorem which is a direct corollary of Sakasai's.

**Theorem 4.3** ([16, Corollary 6.16]). *The group $\mathcal{U}(C_{g,1}^{irr} \cap C_{g,1}[k])$ has an abelian group quotient of infinite rank for $k \geq 2$.*

*Proof.* Let $\gamma \in \pi_{k-1} \setminus \pi_k$ and let $K \subset S^3$ be a tame knot. By Lemma 4.1 we see $M(\gamma, K) \in C_{g,1}^{irr} \cap C_{g,1}[k]$. By Proposition 4.2 we have

$$c \circ \tau_k(M(\gamma, K)) = [\Delta_K(\gamma)].$$

Since it is well-known that for any $p \in \mathbb{Z}[t, t^{-1}]$ with $p(t^{-1}) = p(t)$ and $p(1) = 1$, there exists a knot $K \subset S^3$ such that $\Delta_K = p$, the image of $e \circ c \circ \tau_k\colon C_{g,1}^{irr} \cap C_{g,1}[k] \to \oplus_{[p]}\mathbb{Z}$ contains a submonoid isomorphic to $\mathbb{Z}_{\geq 0}^\infty$. Therefore the image of the induced map $\mathcal{U}(C_{g,1}^{irr} \cap C_{g,1}[k]) \to \mathbb{Z}^\infty$ is a free abelian group of infinite rank, which proves the theorem. □

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